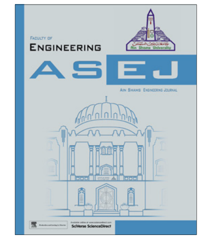
Ain Shams Engineering Journal (2013) 4, 365–368

Ain Shams University

Ain Shams Engineering Journal

www.elsevier.com/locate/asej
www.sciencedirect.com



CIVIL ENGINEERING

Analytical solution to problems of hydraulic jump in horizontal triangular channels

I.M.H. Rashwan

Irrigation and Hydraulic Engineering Department, Faculty of Engineering, Tanta University, Tanta, Egypt

Received 21 May 2012; revised 21 October 2012; accepted 4 November 2012
Available online 29 December 2012

KEYWORDS

Analytical solutions;
Depth ratio;
Ditch irrigation;
Hydraulic jump;
Triangular channels

Abstract A hydraulic jump is formed in a channel whenever supercritical flow changes to subcritical flow in a short distance. It can be used in triangular ditch irrigation to raise the downstream water surface. The basic elements and characteristics of the hydraulic jump are provided to aid designers in selecting more practical basins. In the present study, the slope side, discharge and the energy loss in hydraulic jump in horizontal triangular section are known whereas one has to obtain the sequent depths. The specific force and specific energy equations in a horizontal triangular open channel are made dimensionless, writing it for the sequent depths as a function of discharge and head loss. The proposed modes for hydraulic jump elements are of high accuracy and applicable to a wide range of discharge intensity values and initial conditions without any limitations for the assumptions under consideration.

1. Introduction

The hydraulic jump in triangular open channel has not received much attention. Thus, relatively scarce literature on hydraulic jumps in triangular channels is available to date. Chow [1] declared that the hydraulic jump first investigated by Bidone, an Italian, in 1818. He mentioned that Belanger obtained the explicit solutions of sequent depth ratio for rectangular and prismatic channels without bed friction. Yin [2] presented two simple solution charts for obtaining the change in depth of water passing through a hydraulic jump in circular, triangular, trapezoidal and rectangular channels. In 1987, Hager and Wanoschek [3] declared that, regarding the sequent ratio and the relative energy dissipation, trapezoidal and particularly triangular channels are much more effective than rectangular channels, provided the inflow Froude number F_1 is fixed.

Larry [4] classified ditches (roadside and median channel) that collect and convey storm water from pavement surface, roadside and median areas as open channel systems. He concluded that the design and analysis the ditches follows the basic principles of open channel flow. Young and Stein [5] categorized gutter sections as conventional or shallow type. They used the conventional gutters which have a uniform cross slope with triangular section. Achour and Debabeche [6] showed that the hydraulic jump may be used in triangular ditch irrigation to raise the downstream water surface. Swamee and Rathie [7] demonstrated that, a stilling basin provide

E-mail address: imh_rashwan@yahoo.com

Peer review under responsibility of Ain Shams University.

Production and hosting by Elsevier

2090-4479 © 2012 Ain Shams University. Production and hosting by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.asej.2012.11.007

downstream a weir to dissipate surplus energy by hydraulic jump.

Das [8] developed a methodology for simultaneous determination of alternate depths and sequent depths in trapezoidal, rectangular and triangular channels. This iterative methodology uses solutions of quadratic and cubic equations to identify the two subcritical and supercritical depths. Vatankhah and Kouchakzadeh [9] used iterative fixed-point method to present solutions of specific energy and specific force equations in open channels with trapezoidal, rectangular and triangular cross-sections. In 2010 Vatankhah [10] gave analytical solution of specific energy and specific force equations for trapezoidal and triangular channels. He solved problem for a given channel geometry and discharge, the subcritical (supercritical) depth is found in terms of the other supercritical (subcritical) depth. Vatankhah and Omid [11] gave direct solution of hydraulic jump in triangular channels. Their research showed steps to reach an acceptable physical analytic solution for sequent depth ratios in horizontal triangular channels.

In the present study, the momentum and energy equations are first converted to the dimensionless form. Thus, the side slope of triangular channel, the discharge and the energy loss in the hydraulic jump are known, whereas one has to obtain the sequent depths. The main objective is to developed equations for hydraulic jump elements of high accuracy and applicable to a wide range of discharge intensity values and initial conditions.

2. Theoretical analysis

The hydraulic jump in open channels is a transitional state from an upstream supercritical to downstream subcritical flow. In this transition, water surface rises abruptly, surface rollers are formed, and intense mixing occurs, air is entrained and usually a large amount of energy is dissipated as shown in Fig. 1. The flow depths upstream and downstream of the jump are called sequent depths. Design of stilling basins requires knowledge of various elements of hydraulic jump such as discharge, sequent depths and energy loss. In the case of low slope triangular channel, the solutions of momentum and energy equations involve tedious methods of trial and error.

In applying the momentum equation to a short horizontal reach of a prismatic triangular open channel, the external force of friction and weight effect of water can be ignored. Thus, the momentum equation for uniform velocity distribution becomes:

$$\frac{Q^2}{gA_1}+A_1\bar{y}_1=\frac{Q^2}{gA_2}+A_2\bar{y}_2 \tag{1}$$

where Q = discharge; A_1 and A_2 = water areas at Sections 1 and 2, respectively; $\bar{y}_1$ and $\bar{y}_2$ = the depths from the water surface to the centroids of the Sections 1 and 2, respectively; and g = the acceleration due to gravity.

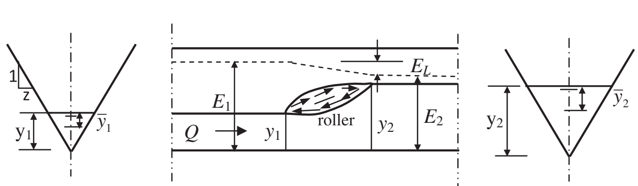


Figure 1 Definition sketch for a hydraulic jump in triangular channel.

For a triangular with side slope z; $A_1=zy_1^2$; $A_2=zy_2^2$; $\bar{y}_1=y_1/3$ and $\bar{y}_2=y_2/3$. Substituting these relations and $y_c=(2Q^2/gz^2)^{1/5}$ in the above equation and simplifying,

$$y_c^5=\frac{2y_2^2y_1^2((y_2+y_1)^2-y_2y_1)}{3(y_2+y_1)} \tag{2}$$

where y_c = critical depth $(2Q^2/gz^2)^{1/5}$, z = side slope; y_1 and y_2 = upstream and downstream water depths; respectively.

Dividing Eq. (2) by critical water depth transformed it to dimensionless equation as:

$$2Y_2^2Y_1^2(Y_2+Y_1)^2-3(Y_2+Y_1)-2Y_2^3Y_1^3=0 \tag{3}$$

where Y_1 = dimensionless upstream water depth (y_1/y_c) and Y_2 = dimensionless downstream water depth (y_2/y_c).

Denoting:

$$\beta=Y_2Y_1 \tag{4}$$

where β = dimensionless parameter.

Substituting β into Eq. (4) gives the following equation:

$$2\beta^2(Y_2+Y_1)^2-3(Y_2+Y_1)-2\beta^3=0 \tag{5}$$

Tacking one square root of Eq. (5) gives the following equation:

$$Y_2+Y_1=\frac{3+\sqrt{9+16\beta^5}}{4\beta^2} \tag{6}$$

The minus sign (the second root) is invalid as $\beta>0.0$.

Solving the two Eqs. (4) and (6) for sequent depths, Y_1 and Y_2, in term of β only leads to the following equations:

$$Y_1=\frac{1}{2}\left[\frac{3+\sqrt{9+16\beta^5}}{4\beta^2}-\left(\left(\frac{3+\sqrt{9+16\beta^5}}{4\beta^2}\right)^2-4\beta\right)^{\frac{1}{2}}\right] \tag{7}$$

$$Y_2=\frac{1}{2}\left[\frac{3+\sqrt{9+16\beta^5}}{4\beta^2}+\left(\left(\frac{3+\sqrt{9+16\beta^5}}{4\beta^2}\right)^2-4\beta\right)^{\frac{1}{2}}\right] \tag{8}$$

2.1. Losses dissipated by the jump

Specific energy may be interpreted as the sum of potential and kinetic energy of fluid with respect to the bottom of the channel. Specific energy before the jump can be written as:

$$E_1=y_1+\alpha_1\frac{Q^2}{2gA_1^2} \tag{9}$$

where E_1 = specific energy at Section 1 (initial specific energy) and α_1 = energy coefficient at Section 1.

Specific energy after the jump expressed as:

$$E_2=y_2+\alpha_2\frac{Q^2}{2gA_2^2} \tag{10}$$

where E_2 = specific energy at Section 2 (sequent specific energy) and α_2 = energy coefficient at Section 2.

For uniform velocity distribution, $\alpha_1=\alpha_2\cong1.0$. The head losses by the jump can be written as:

$$E_L=E_1-E_2=\frac{(y_2-y_1)^3}{6y_2^2y_1^2}((y_2+y_1)^2+y_2y_1) \tag{11}$$

where E_L = head loss.

Eq. (11) can be transformed as follows equation for a triangular cross-section as:

$$\overline{E}_L = \frac{(Y_2 - Y_1)^3}{6(Y_2 Y_1)^2}((Y_2 + Y_1)^2 + Y_2 Y_1) \tag{12}$$

where $\overline{E}_L$ = dimensionless head loss (E_L/y_c).

Substituting β and Eq. (6) into Eq. (12), $\overline{E}_L$ in term of β is obtained as:

$$\overline{E}_L = \left(\frac{(3+\sqrt{9+16\beta^5})^2 + 16\beta^5}{96\beta^6}\right)\left[\left(\frac{3+\sqrt{9+16\beta^5}}{4\beta^2}\right)^2 - 4\beta\right]^{\frac{3}{2}} \tag{13}$$

Eq. (13) is shown plotted in Fig. 2.

An equation was developed based on the data in Fig. 2 using the least-square technique as follows:

$$\beta = 0.8606 - 0.036Ln(\overline{E}_L), \quad R^2 = 0.8866 \tag{14}$$

Eq. (14) is approximate relation between β and energy loss $\overline{E}_L$ and has deviates less than 8.0% from the exact expression.

For preliminary designs, the estimating required tail water depth, length of the jump and loss of energy which can be determined from special design curves for rectangular section [1,4]. For design, the head loss in triangular channels can be assumed as percentage of critical head. Accordingly, the critical head equation for triangular section is:

$$\overline{E}_c = 1.25 \tag{15}$$

where $\overline{E}_c$ = dimensionless critical head (E_c/y_c).

3. Proposed method

The basic elements and characteristics of a hydraulic jump on horizontal aprons are provided to aid designers in selecting more practical basins. For preliminary designs, the estimating loss of energy can be determined as percentage of critical energy, Eq. (15). The hydraulic jump problem can be solved by using momentum and specific energy equations which involve tedious methods of trial and error. In the present study, the side slope, the discharge and the energy loss in hydraulic jump in horizontal triangular section are known whereas one has to obtain the sequent depths. The specific force and specific energy equations in a horizontal triangular open channel are made dimensionless, writing it and reduced to equations for sequent depths, Eqs. (7) and (8). For a given data for the problem are: the side slope, z, the discharge, Q and the energy loss, E_L. The method of analytical solution for sequent depths is illustrated by the following steps:

(1) Compute the critical water depth, $y_c = (2Q^2/gz^2)^{1/5}$.
(2) Using Eq. (13) by trial and error to get β from knowing (estimation) dimensionless head loss $\overline{E}_L = (E_L/y_c)$.
(3) Compute the relative sequent depths $Y_1 = (y_1/y_c)$ and $Y_2 = (y_2/y_c)$ from Eqs. (7) and (8) and then calculate the sequent depths y_1 and y_2.

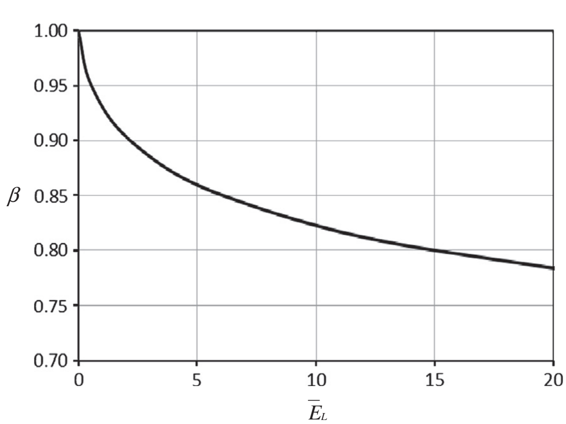


Figure 2 Plot of Eq. (13).

4. Practical examples

For a triangular open channel with side slope z = 1.0, discharge Q = 4.0 m^3/s and head loss as estimation value for design (75% $\overline{E}_c$), it is required to get the sequent depths.

(1) Compute the critical water depth, $y_c = (2Q^2/gz^2)^{1/5}$ = 1.266 m.
(2) Compute the head dimensionless head loss = 0.75 × 1.25 = 0.9375 from Eq. (15).
(3) Using Eq. (13) by trial and error to get β = 0.933 from knowing dimensionless head loss $\overline{E}_L = (E_L/y_c)$ = 0.9375.
(4) Compute the relative sequent depths Y_1 = 0.5995 and Y_2 = 1.5566 from Eqs. (7) and (8), then calculate the sequent depths y_1 = 0.759 m and y_2 = 1.97 m.

5. Conclusions

The equations of sequent depths $Y_1 = y_1/y_c$ and $Y_2 = y_2/y_c$ for hydraulic jump in horizontal triangular open channels are solved with dimensionless variable β, which is a function in the dimensionless independent variable $\overline{E}_L = E_L/y_c$. By introducing dimensionless parameters β into the derived Eqs. (7) and (8) the sequent depths can be accurately calculated for triangular cross-section. The proposed equations for hydraulic jump elements are of high accuracy and applicable to a wide range of discharge intensity values and initial conditions for the assumptions under consideration in comparison to other methods attempted so far.

References

[1] Chow VT. Open channel hydraulics. McGraw Hill; 1959.
[2] Au-Yeung Yin. Solution of hydraulic jump in horizontal channels by graphs. J Hydraulic Div, ASCE 1972;98(8):1465–8.
[3] Hager WH, Wanoschek R. Hydraulic jump in triangular channel. J Hydraulic Res 1987;25(5):549–65.
[4] Mays Larry W. Hydraulic design handbook. McGraw Hill; 1999.
[5] Kenneth Young G, Stuart Stein JR. Hydraulic design book. McGraw-Hill Companies; 2004.
[6] Achour B, Debabeche M. Control of hydraulic jump by sill in triangular channel. J Hydraulic Res 2003;41(1):319–25.
[7] Swamee PK, Rathie PN. Exact solution for sequent depths problem. J Irrigat Drain Eng ASCE 2004;130(6):520–2.

[8] Das A. Solution of specific energy and specific force equations; trapezoidal and triangular channels. J Irrigat Drain Eng, ASCE 2007;134(4):407–10.
[9] Vatankhah Ali R, Kouchakzadeh S. Discussion of solution of specific energy and specific force equations. In: Das A. Journal of irrigation and drainage engineering ASCE; vol. 134(6). 2008. p. 880–82.
[10] Vatankhah Ali R. Analytical solution of specific energy and specific force equations; trapezoidal and triangular channels. Adv Water Res 2010;33:184–9.
[11] Vatankhah Ali R, Omid MH. Direct solution to problems of hydraulic jump in horizontal triangular channels. Appl Math Lett 2010;23:1104–8.

Dr. Ibrahim Mohamed Hussein Rashwan, Associate Professor of Irrigation and hydraulics Engineering, Faculty of Engineering, Irrigation and hydraulics Engineering Department, Tanta University, Tanta, Egypt. The author graduated from Mansoura University and received his M.Sc. Degree in 1979 and 1987, respectively. He obtained his Ph.D. in 1999 from Alexandria University. He is an Engineering Faculty Member at Tanta University since 1991. His research interests comprise: open channel hydraulics; contamination transport and control, water distribution network, dam hydraulics failure and circular open channels.